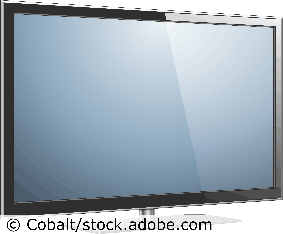
2 Functions

Activity: Linear programming  
(Student version)

Vocabulary

* **Linear programming** is the mathematical process of analysing a system of inequalities to make the best decisions given the constraints of the situation.
* **Constraints (limitations)** are the particular restrictions of a situation due to time, money, or materials.
* The common shaded region of the system of inequalities is called the **feasible region**.
* In an **optimisation (best solution)** problem, the goal is to locate the feasible region of the system and use it to find a maximum value (such as **profit**) or a minimum value, (such as **cost**).

Worked example: Manufacturing

* A small TV manufacturing company produces large and small televisions.
* To build these TVs, they require three different processes using three different machines (A, B and C).
* The table below shows the number of hours that are required on each machine per day in order to produce a large or a small television (**constraints**).
* The ‘hours available’ is how much time the machine is available for use.

|  |  |  |  |
| --- | --- | --- | --- |
| **Machine** | **Time required per large TV /hours** | **Time required per small TV /hours** | **Hours available** |
| A | 1 | 2 | 16 |
| B | 1 | 1 | 9 |
| C | 4 | 1 | 24 |

* Suppose the company receives $60 profit for the large TVs and $40 for the small TVs.

**Question:** How many large and small TVs should be produced each day to maximise profit?

First, establish your variables. In this case we are asked how many of each TV should be made in order to maximise profit.

Assume *x* = large TVs and *y* = small TVs.

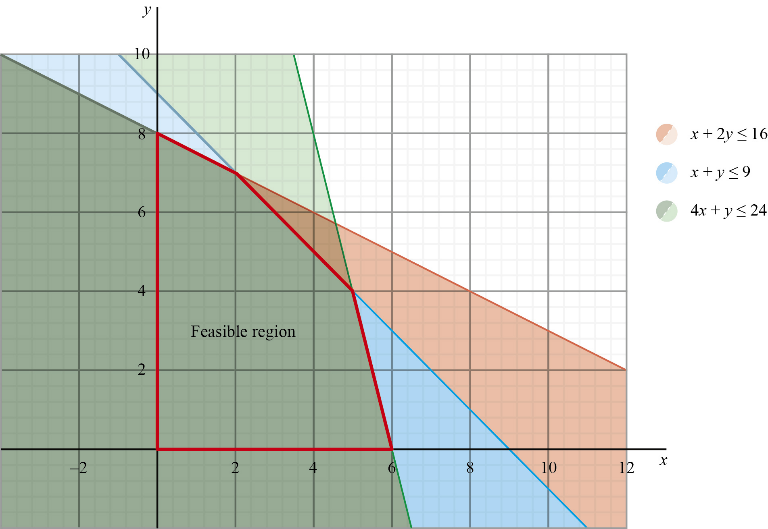
We can then use the information given in the table to generate the following system of inequalities:

Equation A: *x* + 2*y* ≤ 16

Equation B: *x* + *y* ≤ 9

Equation C: 4*x* + *y* ≤ 24

Using your GDC or a graphing website draw the three inequalities on the same set of axes.



This then produces a ‘feasible region’, where all three regions overlap. This means that any coordinate pair taken in this region will fit the constraints for the problem.

For example, the coordinate (4, 5) is in the feasible region. We can test this by using the three equations that we generated.

If *x* = 4 and *y* = 5 then the following results occur:

4 + 2(5) = 14 This works as 14 is less than 16.

4 + 5 = 9 This works because 9 is equal to 9.

4(4) + 5 = 21 This works because 21 is less than 24.

Now we have established our ‘feasible region’, we have to maximise profit.

Take our values *x* = 4 and *y* = 5. This means four large TVs and five small TVs. This would generate a profit of

60(4) + 40(5) = $440

By trialling points from our feasible region, we can see that the maximum profit occurs when *x* = 5 and *y* = 4.

This generates a profit of $460.

Homework problem: Ski manufacturing

* A sporting goods company manufactures two types of skis: a racing model and a freestyle model.
* Each pair of racing skis requires 3 hours of labour and the company produces at most 20 pairs of racing skis per day.
* Each pair of freestyle skis requires 2 hours of labour and the company can produce at most 30 pairs of freestyle skis per day.
* They have a maximum total of 90 hours available for ski production.
* The profit on each pair of racing skis is $30 and $40 on each pair of freestyle skis.

**Question:** How many pairs of each should be manufactured in order to maximise profits? Use problem solving and linear programming techniques to solve.

**Additional question:** Suppose that the profit on each pair of freestyle skis is $20. Show that there are multiple ways to maximise profit.